

The change in crankcase volume A is the volume between the two piston displacements and is a function of d the phase angle between the pistons. Assuming identical cylinder bore diameters then

$$A(d) = \int_l^u (\sin x - \sin(x + d)) dx$$

where l and u are the two solutions of $\sin x = \sin(x + d)$ when $\sin(x + d) \geq \sin(x)$ over the range d ($0 \leq d \leq \pi$)

The solution is

$$A := \cos(a) - \cos(a + d) - \cos(b) + \cos(b + d)$$

where $a = -\arctan(\sin(d)/(-1 + \cos(d)))$ and $b = \pi - \arctan(\sin(d)/(-1 + \cos(d)))$ are the lower and upper limits that satisfy $\sin(a + d) = \sin(a)$ and $\sin(b + d) = \sin(b)$.

This evaluates to

$$A(d) = 2 \frac{1}{\sqrt{1 + \frac{(\sin(d))^2}{(-1 + \cos(d))^2}}} - 2 \cos\left(-\arctan\left(\frac{\sin(d)}{-1 + \cos(d)}\right) + d\right)$$

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